

University of Groningen

The prediction of rotation curves in gas-dominated dwarf galaxies with modified dynamics

Sanders, R. H.

Published in:
Monthly Notices of the Royal Astronomical Society

DOI:
[10.1093/mnras/stz353](https://doi.org/10.1093/mnras/stz353)

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2019

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Sanders, R. H. (2019). The prediction of rotation curves in gas-dominated dwarf galaxies with modified dynamics. *Monthly Notices of the Royal Astronomical Society*, 485(1), 513-521.
<https://doi.org/10.1093/mnras/stz353>

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

The prediction of rotation curves in gas-dominated dwarf galaxies with modified dynamics

R. H. Sanders[★]

Kapteyn Astronomical Institute, P.O. Box 800, NL-9700 AV Groningen, the Netherlands

Accepted 2019 February 1. Received 2019 January 26; in original form 2018 November 13

ABSTRACT

I consider the observed rotation curves of 12 gas-dominated low-surface-brightness galaxies – objects in which the mass of gas ranges between 2.2 and 27 times the mass of the stellar disc (mean = 9.4). This means that, in the usual decomposition of rotation curves into those resulting from various mass components, the mass-to-light ratio of the luminous stellar disc effectively vanishes as an additional adjustable parameter. It is seen that the observed rotation curves reflect the observed structure in gas surface density distribution often in detail. This fact is difficult to comprehend in the context of the dark matter paradigm where the dark halo completely dominates the gravitational potential in the low surface density systems; however it is an expected result in the context of modified Newtonian dynamics (MOND) in which the baryonic matter is the only component. With MOND the calculated rotation curves are effectively parameter-free predictions.

Key words: galaxies: dwarf – galaxies: kinematics and dynamics – dark matter.

1 INTRODUCTION

In a number of late-type dwarf galaxies, the baryonic mass content is dominated by neutral gas and not by luminous stars or molecular gas. In the analysis of rotation curves in terms of various components this simplifies the decomposition of the mass distribution by largely removing the uncertain mass-to-light ratio of the stellar disc as an adjustable parameter. In the context of the standard cold dark matter (CDM) paradigm the dynamics is dominated by the dark matter halo which has a standard (Navarro–Frenk–White) form and the gas motion traces, as test particles, the radial gravitational force within the halo.

Yet, the observed rotation curves show a surprising variety of shapes given the dominance of the purported halo with a standard density distribution (Navarro, Frenk & White 1996). This problem is recognized and a number of solutions in the context of the CDM paradigm have been proposed: the effects of baryon-induced fluctuations due to gas dynamical feedback in the presence of ongoing star formation (Oman et al. 2015); dynamical friction of gas clouds formed by gravitational instability leading to transfer of energy to dark matter (Del Popolo et al. 2018); modification of the presumed properties of dark matter particles so that they are self-interacting (SIDM) and form isothermal halo cores (Ren et al. 2018). Most of these mechanisms were originally designed to solve perceived generic problems such as the observational appearance of cores in galaxies rather than cusps as predicted in pure cosmological N -body simulations (Read et al. 2016). The problem presented

by diversity of shapes has more recently been addressed and is explained by the gravitational influence of baryons on the more responsive halo core as opposed to the harder cusp.

But the problem is more severe than the general issue of the diversity of rotation curves; the form of the observed rotation curves is most often traced in detail by the distribution of baryonic matter. This is particularly true in low-surface-brightness dwarf galaxies where the presumed dark component is overwhelmingly dominant. I argue here that this is a fundamental problem for CDM. Taking a sample of 12 gas-dominated systems, I show that this variety of shapes is directly related to the form of the observable gas, or effectively, the baryonic mass distribution. The diversity of rotation curve shapes follows directly from the diversity in the distribution of baryons, and the relationship in form as well as amplitude is well-described in detail by modified Newtonian dynamics, or MOND, proposed by Milgrom (1983) as an alternative to astronomical dark matter.

In this context MOND may be viewed as an algorithm – a modified Poisson relation between the observable baryonic mass distribution and the mean gravitational acceleration in an astronomical object. Basically the idea is that the true gravitational acceleration g is related to the Newtonian acceleration g_N (the derivative of a potential given by the Newtonian Poisson equation) by

$$g\mu(g/a_0) = g_N, \quad (1)$$

where a_0 is a universal constant with units of acceleration and $\mu(x)$ is an unspecified function that interpolates between the Newtonian

[★] E-mail: sanders@astro.rug.nl

Table 1. Gas-dominated galaxies.

Galaxy	D Mpc	V_{rot} km s ⁻¹	Incl. kin. deg	Incl. ph. deg	M_{gas} 10 ⁸	M_{stars} M _⊙	$M_{\text{gas}}/M_{\text{stars}}$ 10 ⁸ M _⊙	Refs. D & Incl.
DDO 52	10.3	60	43	51	3.3	0.72	4.6	1
DDO 87	7.7	55	56	59	2.9	0.62	4.7	1
DDO 126	4.9	38	65	68	1.6	0.23	7.2	1
DDO 133	3.5	46	43	49	1.3	.26	4.9	1
DDO 154	3.7	51	68	65	3.5	0.13	27.	1
IC 2574	3.2	66	77	83	8.1	0.67	16.	2
NGC 3741	3.2	50	64	57	2.0.	0.087	23.	3
Har 29	5.9	34	61	59	0.94	0.14	6.5	1
UGC 4499	13.9	75	50	47	15.0	4.5	3.1	4
UGC 5005	53	99	40	41	44.	4.7	8.5	4
UGC 5750	59	79	61	70	20.	9.3	2.2	4
WLM	1.0	35	74	70	0.80	0.16	4.9	1

Note. (1) Oh et al. 2015; (2) Sanders 1996; (3) Gentile et al. 2007; (4) Swaters, et al. 2010.

regime ($x \gg 1$, $\mu(x) = 1$) and the low acceleration regime ($x \ll 1$, $\mu(x) = x$).

There are well-known observational and theoretical problems with non-relativistic MOND on scales larger than galaxy groups (but see Milgrom 2018): not accounting for the full discrepancy in rich clusters of galaxies; absence of a consistent cosmology or cosmogony; no definite prediction of the form of anisotropies in the cosmic background radiation – all problems that are addressed by the paradigm of CDM. MOND still requires a consistent relativistic extension, but the fact that $a_0 \approx cH_0/6$ is suggestive of a cosmological connection. While these are issues that must be addressed by a full theory, they do not subtract from the phenomenological success of MOND in the treatment of galaxy rotation curves (see Famaey and McGaugh 2012 for an up-to-date review of the observational and theoretical status of MOND.).

An advantage of considering this sample of dwarf galaxies is that the objects are mostly in the low acceleration regime and the exact form of the interpolating function does not play a significant role, i.e. $g \approx \sqrt{g_N a_0}$. Moreover, the distribution of the baryonic surface density, the gas, is directly observed. Thus, the fact that the mass-to-light ratio of a stellar component vanishes as an adjustable parameter means that the MOND rotation curves are essentially parameter-free predictions: *MOND predicts the form and amplitude of the rotation curves from the observed distribution of baryons with only one additional universal parameter having units of acceleration.* And it works well in most cases.

The existence of an algorithm that permits the prediction of rotation curves from the observed distribution of baryonic matter is extremely challenging to the dark matter paradigm because it is not evidently a property permitted by a non-interacting, non-dissipative medium (also SIDM). Until any dark matter model can achieve comparable predictive success with one additional fixed parameter, the CDM model cannot be considered on a par with MOND on the scale of galaxies.

2 THE SAMPLE

The objects considered are listed in Table 1. Seven of these are from the HI observations of dwarf irregular spiral galaxies, the ‘little things’ (LT) survey (Hunter et al. 2012). These are galaxies with maximum observed rotation velocities in the range of 30 to 60 km s⁻¹ and gas (neutral hydrogen plus primordial helium) to stellar mass ratios ranging from five to 30; in other words, they are extremely gas-dominated systems. The remaining five are from

the literature including three from the sample of dwarfs described by Swaters, Sanders & McGaugh (2010). These also are highly gas-dominated objects ranging up to gas-to-star mass ratios in excess of 20.

For the dwarf galaxies there are well-known difficulties presented by the use of two-dimensional velocity fields to determine the run of circular velocity. These galaxies are, after all, irregular which means that they are generally asymmetric in both their light and gas distributions as well as their kinematics. For the LT galaxies these problems are particularly severe. The asymmetry results in a basic uncertainty in the projection of the measured density or velocity fields as embodied primarily by the inclination parameter (Oh et al. 2015).

By what factors should the velocity fields be de-projected? Typically, in such analyses, the average inclination is restricted to be greater than 50° but less than 80° to reduce ambiguities resulting from de-projection. But only 15 of the 26 objects in the LT sample of Oh et al. meet this criterion in terms of their adopted global inclinations. Moreover, there is also a large scatter of fitted inclinations in the kinematic tilted ring analysis of single objects (see DDO 70 for example, given in Oh et al.). The irregular light and gas distributions, and in several cases substantial differences between the photometric and kinematic inclinations, suggest the presence of basic structural asymmetries (bars for example) that can lead to large-scale and un-modelled non-circular motions. We should bear in mind that all of these effects can call into question the role of the published rotation curves as an accurate tracer of the underlying gravitational force (also see the appendix below).

To minimize these difficulties the seven LT galaxies included here all satisfy the following criteria: (1) all global inclinations lie between 40° and 80°; (2) the difference between the kinematic and visual inclinations is less than 10° (this reduces the possibly significant contribution of bars to the velocity fields.); (3) objects with strongly asymmetric velocity fields (see Iorio et al. 2017) are not included (this is also indicated by a highly variable and fluctuating inclination parameter in the tilted ring modelling.).

In what follows I fix all distances, inclinations and stellar masses at their nominal values in the given references. For example, in all of the LT galaxies these parameters are taken as given in Oh et al. For the UGC dwarfs the values are taken from Swaters et al. This means that there is no tweaking of parameters to achieve a rotation curve that more closely agrees with the observations. Thus the procedure is more in the spirit of prediction and not fitting, but one should keep in mind that the expectations should be somewhat lower than in the case where parameters may slide.

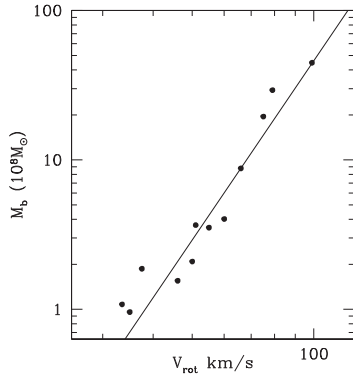


Figure 1. The baryonic Tully–Fisher relation defined by the sample of gas-rich dwarfs. The contribution of the stellar mass is included as specified in Table 1. The line is the MOND prediction ($V^4 = Ga_0 M$ with $a_0 = 1.2 \times 10^{-10} \text{ m s}^{-2}$).

The baryonic mass–rotation velocity relation (Tully & Fisher 1977, McGaugh 2005) described by these gas-rich dwarfs is shown in Fig. 1. Here solid line is not a fit but is the relation predicted by MOND, $v^4 = Ga_0 M_b$, where $a_0 = 1.2 \times 10^{-10} \text{ m s}^{-2}$ as usual. This immediately demonstrates that the amplitude of the rotation curves is directly related to the mass of baryonic matter in the sample gas-rich galaxies in the same manner as in objects with higher rotation velocities where the mass is dominated by the stellar disc (McGaugh 2005).

3 PREDICTED ROTATION CURVES

In calculating predicted rotation curves I take the Newtonian rotation curves of the baryonic components, assumed to be in a thin disc, as given in the indicated references (Table 1, column 9). I convert to the MOND rotation curves by applying equation (1) where the interpolating function is assumed to be of the ‘standard’ form ($\mu(x) = x/\sqrt{1+x^2}$). As noted above for the low surface density objects (low acceleration) the calculated rotation curves are relatively insensitive to the exact form. (In all cases, the calculated rotation velocities are within a few per cent of that given by the asymptotic form of $\mu(x) \approx x$).

For the sample galaxies, the resulting rotation curves predicted by Newton (dashed curves) and by MOND (solid curves) are compared with the observations in Fig. 2 (lower panels), along with the surface density distributions of the baryonic components. It is striking that in several cases (DDO 87, WLM, IC 2574, NGC 3741) the observed structure corresponds quite precisely to that predicted by the baryonic mass distribution in the context of MOND. Note that for WLM, there are a large number of points on the derived rotation curve given by Oh et al. These are certainly not independent, and given their high density obscure the match of the predicted rotation curve to the observations; therefore I have taken only every fourth point in the curve presented here.

The claim here is that these rotation curves are essentially predictions with no free parameters because of the dominance of the gaseous component. But of course there is some contribution from the luminous disc so one might question the extent to which M/L of the disc does vanish as an adjustable parameter. The stellar masses of the sample galaxies (column 6 in Table 1) are taken from fitting the colours of the galaxies to population synthesis models in the cases of the seven LT galaxies (Oh et al. 2015). For the other five cases they are taken from kinematic considerations (e.g. not

exceeding the observed rotation velocity). In half the cases, the implied stellar masses exceed 15 per cent of the total baryonic mass (those objects in which the ratio of gas mass to stellar mass is less than six, column 8). This can certainly affect the shape of the galaxy rotation curves, especially in the inner regions where the stellar disc often contributes more significantly to the baryonic surface density distribution.

In Fig. 3 we see the calculated rotation curves for one case where the fraction of stellar mass is fairly typical, DDO 87. The lower dashed curve is the Newtonian rotation curve of baryons excluding the estimated contribution of the visible disc and the upper dashed curve is the same but including the stellar disc with a contribution of 17.5 per cent of the total baryonic mass as given in Table 1. The upper and lower solid curves are the same but for the MOND rotation curves. While it is obvious that the contribution of the stellar disc provides better agreement with the observations, it is also clear that, with no stellar contribution at all, the rotation curve is qualitatively similar: The overall form of the calculated rotation curves is quite independent of the mass-to-light ratio of the stellar disc and, within the errors, is consistent with the final observed asymptotic rotation velocity.

The case of an extremely gas-rich galaxy, NGC 3741, is shown in Fig. 4 where again we see the calculated rotation curves with and without the contribution of the stellar disc compared to the observations (Gentile et al. 2007). For this object the gas disc extends 40 exponential scale lengths beyond the luminous disc and is 23 times more massive; i.e. the stellar disc is roughly 4 per cent of the total baryonic mass and, based upon $M/L_K = 0.3$, is consistent with stellar population synthesis. The calculated rotation curves, with and without stars, agree in the outer regions but deviate somewhat in the inner regions where the stellar disc contributes. The observed rotation curve suggests the presence of two components, and in the MOND prediction these are the two baryonic components: the central luminous disc and extended neutral gas disc. But the overall agreement of the calculated rotation curves with and without stars demonstrates the independence of the calculated rotation curve from the M/L of the stellar disc.

An additional factor that can affect the prediction of rotation curves is the uncertainty of the distance estimate – particularly true in an acceleration-based modification such as MOND. For most of these objects, in particular those closer than 10 Mpc, the distance is estimated via the ‘tip of the red giant branch’ method (TRGB) which is generally thought to be accurate to within 5 per cent. Therefore I take these distances as given in the literature, primarily by Hunter et al. (2012) for the LT galaxies. But one should bear in mind that there can be systematic differences in the distance scaling between these nearby objects and more distant objects with Hubble law determinations. Moreover, there are indications that the reported accuracy may be optimistic. For example, in the case of DDO 126 the estimated distances range from 3.9 to 5.1 Mpc (Karachentsev et al. 2003), all of these relying upon the same method (TRGB).

For DDO 126 the shape is reasonably well matched but the predicted amplitude is too high. Given the range of estimates for the distance we see that this could be due to an overestimate of the distance to the galaxy. Fig. 5 shows the predicted rotation curves for the preferred distance of Oh et al., 4.9 Mpc (upper solid curve), as well as the lowest value cited by Karachentsev et al., 3.9 Mpc (the lower solid curve). In the case of the shorter estimate the agreement of the MOND prediction with the observed curve is more precise. For these galaxies in general the predicted rotation curve agrees with that observed to within the likely uncertainty

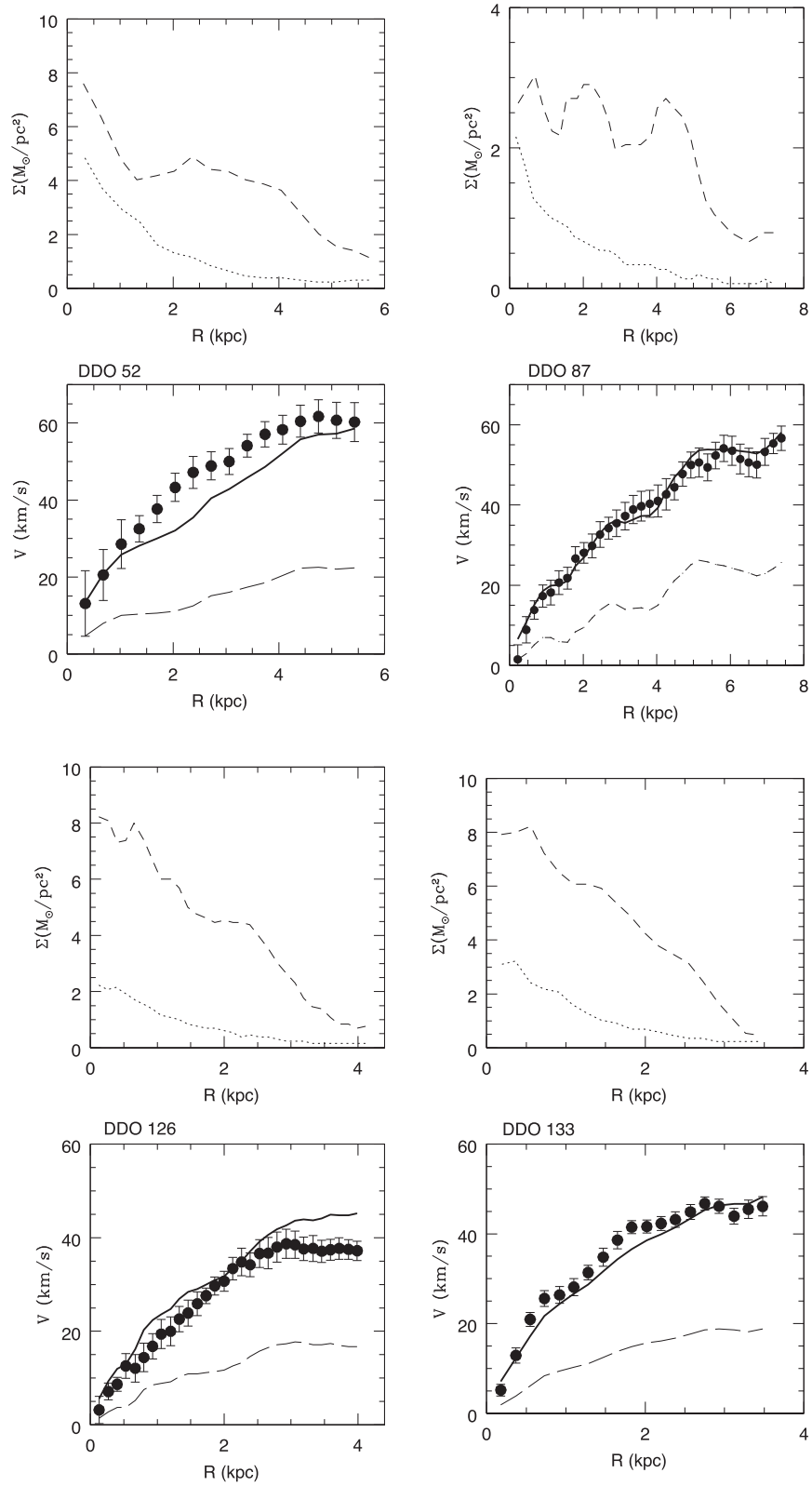


Figure 2. Each galaxy is represented in two panels. The upper panel is the surface densities of gas (dashed) and stars (dotted) as a function of radius. The lower panel shows the observed rotation curve (points), the Newtonian rotation curve of baryonic components (long dashed curve), and MOND rotation curve, calculated from the Newtonian curve via equation (1) (long dashed curve).

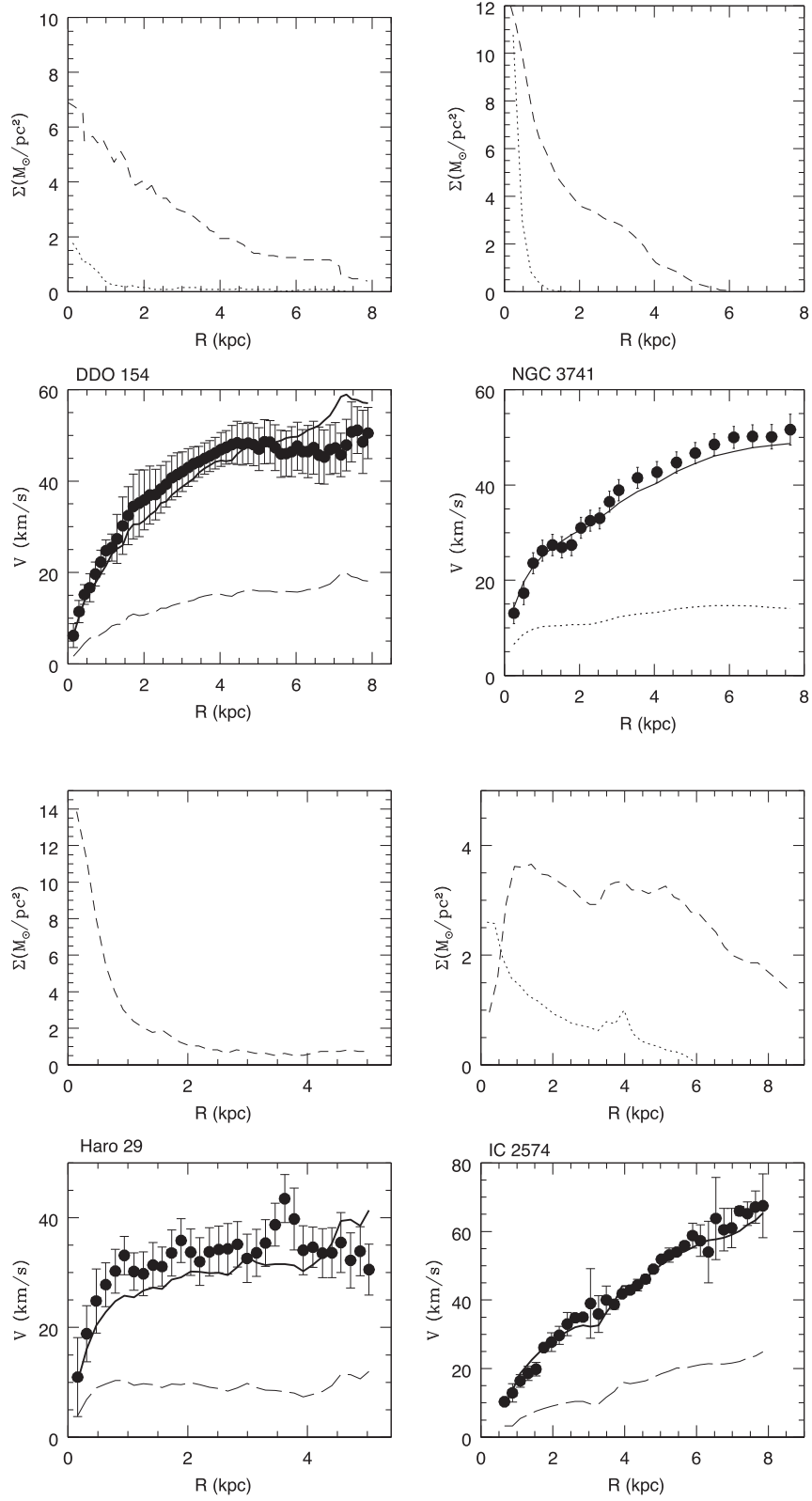


Figure 2 – continued

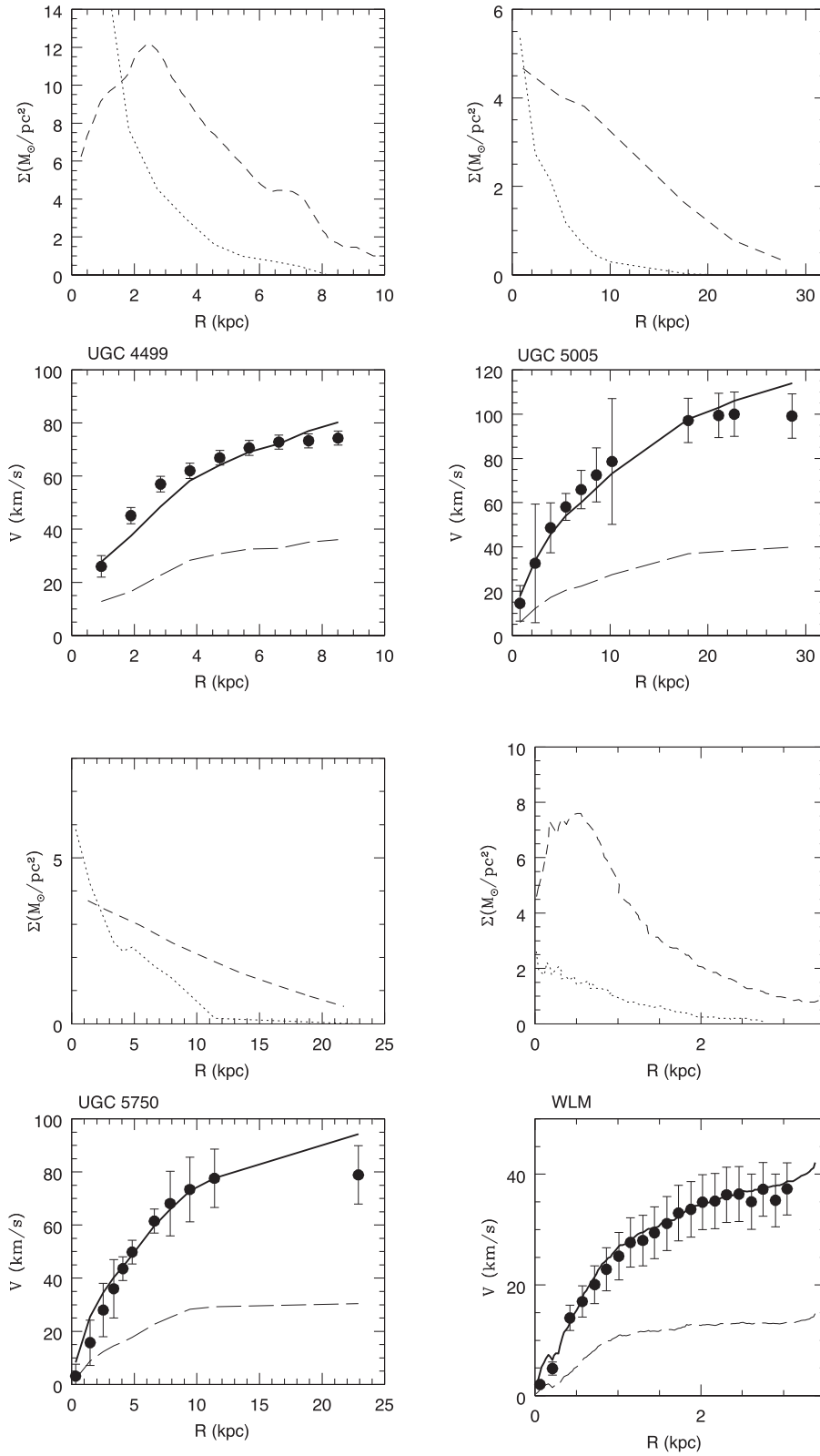


Figure 2 – continued

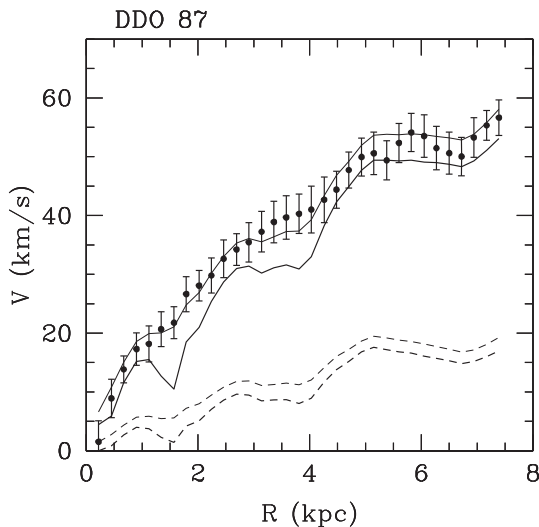


Figure 3. Predicted rotation curves of DDO 87, with and without stars. The dashed lines are the Newtonian rotation curves and the solid curves are the MOND rotation curves. For both Newton and MOND the lower curves are with only the gaseous component and no stellar component (17.5 per cent of total baryonic mass). The upper curves are calculated with the total baryonic mass.

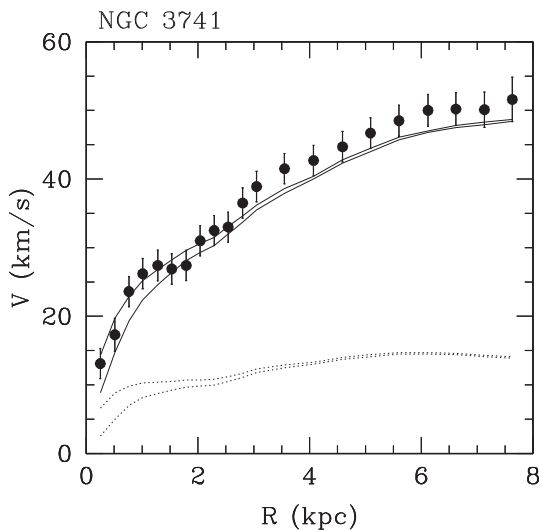


Figure 4. Predicted rotation curves of NGC 3741 with and without stars. As above, the dashed lines are the Newtonian curves and the solid curves are the MOND rotation curves. In both cases the lower curves include only gas with no stellar component (4 per cent of total baryonic mass) and the upper curves are calculated with the total baryonic mass.

introduced by the distance and inclination. This illustrates that the given error bars do not capture the systematic effects introduced by the uncertainty in the fundamental parameters of distance and projection.

4 CONCLUSIONS

MOND predicts an observed phenomenology in galaxies that has only recently been considered by dark matter theorists. For example, there is the ‘radial acceleration relation’ (RAR), a precise universal relationship between the baryonic Newtonian acceleration and the

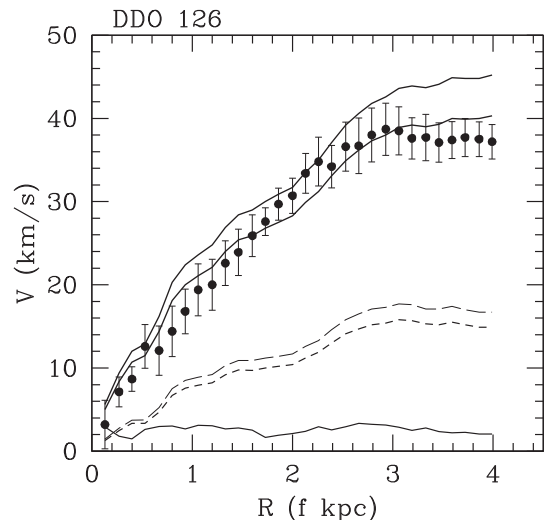


Figure 5. The effect of distance estimates on the predicted rotation curve of DDO 126. Solid curves are the MOND curves for two different distance estimates: the upper curve for the distance adopted by Oh et al. (4.9 Mpc) from Karachentsev et al. (2003). The lower solid curve is taking the lowest estimate of the distance referenced by Karachentsev et al. (3.9 Mpc). For this lower value the units of radius (x-axis) are scaled by the ratio of the distances ($f = 0.8$). The dashed curve is the Newtonian curve of the baryonic components.

measured centripetal acceleration in spiral galaxies (McGaugh, Lelli & Schombert 2016). The RAR is subsumed by MOND (as in equation 1), but only recently, after being reported in this cogent observational form, has been considered in the context of dark haloes. It would have been more impressive if the RAR had been predicted a priori as with MOND; the mining of data post facto to probe the validity of theories is plagued by faulty conclusions built on complicated modelling. To match MOND phenomenology the recent dark halo modelling is contrived to reproduce the properties of MOND, most notably a characteristic acceleration. However, the fact remains that in the haloes that form in cosmological N -body simulations (even with baryonic ‘repairs’) no characteristic *fixed* acceleration emerges.

In the context of dark matter haloes observed rotation curves can only constrain the properties of the halo; the exercise is one of ‘fitting’ free parameters (usually three) of a dark matter halo model and luminous disc to the observed rotation curve. Given the flexibility inherent in adjusting the density distribution of an unseen halo (and contriving mechanisms such as SIDM for such adjustments) a fit is always possible. It would seem that the paradigm cannot be falsified by these direct observations of the force distribution in halo-dominated astronomical objects.

The essential point here is that the role of MOND in addressing observed galaxy rotation curves is fundamentally different from that of dark matter. MOND, as considered here, is an algorithm for calculating the rotation curves of spiral galaxies from the observed distribution of baryonic matter. It is an inherently predictive and not a fitting algorithm. While this is particularly evident where the distribution of baryonic matter is the directly observed distribution of neutral gas, it is also evident in more luminous galaxies in which the stellar disc dominates the mass distribution. It was demonstrated years ago (Sanders & McGaugh 2002) that with MOND the fitted disc M/L_B values as a function of colour are consistent with stellar

population models; MOND has no way of knowing that redder discs should have higher blue-band mass-to-light ratios.

Of course, dark halo fits to rotation curves can appear to be more precise because they are fits. There is no flexibility in MOND as opposed to dark matter haloes, and this rigidity should be seen as an advantage. MOND predicts the rotation curves of disc galaxies (and the RAR and the slope and scaling of the Tully–Fisher relationship) with a simple formula containing one new universal parameter – an acceleration with an apparent cosmological significance. Intricate multi-parameter model fitting is not required. And given the uncertainties inherent in converting a two-dimensional radial velocity field to a circular velocity rotation curve in irregular galaxies, we see that MOND has a surprising degree of success.

This fact in itself constitutes a severe challenge to the dark matter paradigm because it is not obviously a property allowed by dark matter haloes as they are perceived to be – consisting of a dominant fluid of undetected particles of unknown nature and interacting with baryonic matter primarily by gravitation. The phenomenology of galaxy rotation curves is tied in detail to the distribution of the assumed sub-dominant baryonic component and the appearance of a ‘dark matter’ discrepancy occurs below a fixed acceleration. It remains unclear how dark matter can mimic this phenomenological result.

ACKNOWLEDGEMENTS

I am grateful to Moti Milgrom for his characteristically incisive comments on the original manuscript.

REFERENCES

- Begeman K. G., 1987, PhD thesis. Kapteyn institute
 Del Popolo A., Pace F., Le Delliou M., Lee X., 2018, *Phys. Rev. D*, 98, 063517
 Famaey B., McGaugh S. S., 2012, *Living Rev. Relativ.*, 15, 10
 Gentile G., Salucci P., Klein U., Granato G. L., 2007, *MNRAS*, 375, 199
 Hunter D. A. et al., 2012, *AJ*, 144, 134
 Iorio G., Fraternali F., Nipoti C., Di Teodoro E., Read J. I., Battaglia G., 2017, *MNRAS*, 462, 3628
 Karachentsev I. D. et al., 2003, *A&A*, 398, 467
 McGaugh S. S., 2005, *ApJ*, 632, 859
 McGaugh S. S., Lelli L., Schombert J. M., 2016, *Phys. Rev. Lett.*, 117, 1101
 Milgrom M., 1983, *ApJ*, 270, 365
 Milgrom M., 2018, preprint ([arXiv:1810.03089](https://arxiv.org/abs/1810.03089))
 Navarro J. F., Frenk C. S., White S. D. M., 1996, *ApJ*, 462, 563
 Oh S.-H. et al., 2015, *AJ*, 149, 180
 Oman K. A. et al., 2015, *MNRAS*, 452, 3650
 Read J. I., Iorio G., Agertz O., Fraternali F., 2016, *MNRAS*, 462, 3628
 Ren T., Awa A., Kaplinghat M., Yu H.-B., 2018, preprint ([arXiv:1808.05696](https://arxiv.org/abs/1808.05696))
 Rogstad D. H., Shostak G. S., 1971, *A&A*, 13, 99
 Sanders R. H., 1996, *ApJ*, 473, 117
 Sanders R. H., McGaugh S. S., 2002, *Ann. Rev. Astron. Astrophys.*, 40, 263
 Swaters R. A., 1999, PhD thesis. Kapteyn Institute
 Swaters R. A., Sanders R. H., McGaugh S. S., 2010, *ApJ*, 718, 380
 Tully R. B., Fisher J. R., 1977, *A&A*, 54, 661

APPENDIX A: OBSERVED ROTATION CURVES OF DWARF IRREGULAR GALAXIES: THE EFFECTS OF METHODOLOGY

The data provided by synthesis radio telescopes are in the form of three-dimensional data cubes for each object: two spatial dimensions on the plane of the sky and one spectral dimension (line emission as a function of radial velocity). Traditionally a

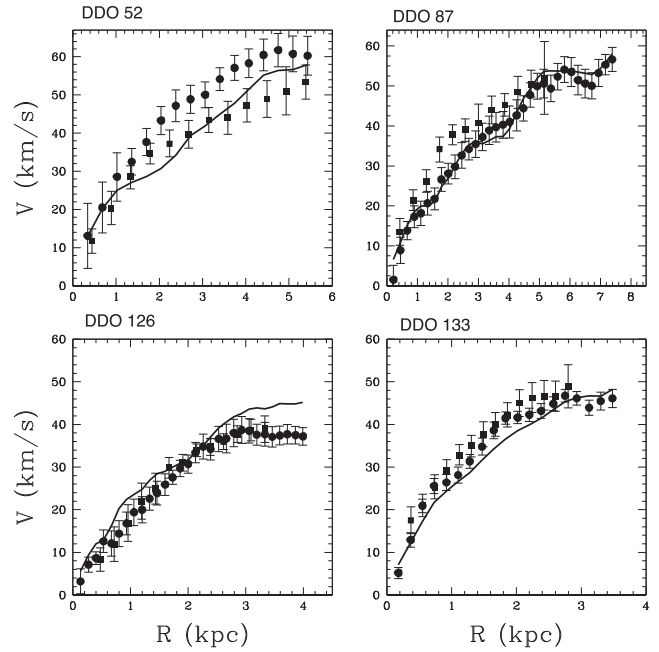


Figure A1. Four galaxies from the present sample with rotation curves determined by the traditional method – derivation of a two-dimensional velocity field (round points) – and by model fitting to the entire three-dimensional data cube (square points). The MOND predicted rotation curves as in Fig. A1 are also shown.

two-dimensional velocity field is first derived by determining a characteristic velocity at each spatial point from the line profile by one of several methods; e.g. an intensity weighted mean or fitting Gaussians (Rogstad & Shostak 1971, Begeman 1987, Oh et al. 2015). A tilted ring model (free parameters are the inclination, position angle and rotation velocity of individual rings) is then fitted to this two-dimensional velocity field to derive the run of circular velocity with radius. This method generally works well when the galaxy is large (compared to the beam) and regular and with no significant distortions due to non-circular motion or noise. But these two conditions are often not satisfied in the case of small dwarf irregular spirals with noisy asymmetric velocity fields.

An alternative to this approach is to directly model the full three-dimensional data cube with no explicit extraction of a two-dimensional velocity field (Swaters 1999). That is to say, the tilted ring model is fit directly to this full three-dimensional cube. Moreover, after convolving the three-dimensional data with the instrumental response, it is found that the derived rotation curves are generally less affected by the finite beam.

Iorio et al. (2017) have applied this method to the little ‘things’ galaxies which make up the majority of the sample considered here (seven out of 12), and have shown that in several cases there are significant differences between the rotation curves derived by the two methods (i.e. greater than the combined error bars). Rotation curves of four of the sample galaxies, derived by both methods, are shown in Fig. A1. Here the round points indicate the rotational velocity determined by the standard method (deriving first a two-dimensional velocity field; Oh et al. 2015), and the square points show the results obtained by fitting to the three-dimensional data cube (Iorio et al. 2017). The predicted rotation curves (as in Fig. 2) are also shown.

In the top two panels, DDO 52 and DDO 87, there are significant differences between the rotation curves derived by the two different methods, especially for DDO 52. For this object the MOND prediction is clearly in better agreement with the curve derived by the second method. For DDO 87 the differences are less pronounced but the predicted curve corresponds more closely to the curve derived by the first method, particularly with respect to the detailed structure.

In the lower two panels, DDO 126 and DDO 133, the results provided by the two different methods do not differ significantly, and the agreement of the MOND prediction is also, of course, similar in quality (for the other two objects in the present sample that were also considered by Iorio et al., DDO 154 and WLM, there

are no significant differences between the results given by the two different methods.). Basically, the difficulty is that these objects are problematic with respect to defining the circular velocity because of intrinsic projection uncertainties introduced by irregularities and asymmetries. Therefore, the quoted error bars on the circular velocity should not be taken too seriously, but the three-dimensional fitting method does, as Iorio et al. claim, provide a more robust estimate of the run of circular velocity. In either case, the MOND predictions, determined from the observed distribution of baryonic matter, yield an acceptable match to the derived rotation curves.

This paper has been typeset from a \LaTeX file prepared by the author.